A diagram of a free-body diagram

Description automatically generatedMass spring systems

We are going to take to be as 0. This also makes the solution much easier as the differential equation becomes homogenous.

Here we are assuming that the roots are distinct and real, so we will use the general solution with e to the specific roots, when the roots are distinct and real, the damping is heavy.

Below are the initial conditions for displacement and velocity, initial time will be set to 0.

First we look at initial displacement,

Now we look at initial velocity,

Now we input t = 0, as we are considering initial velocity to be the velocity of the mass as soon as the simulation starts,

Now that we have found A, we can now find B –

So here are the A and the B,

So, the particular solution to the differential equation shown in the first page would follow this -

The above equation is valid for heavy damping, for oscillations with underdamping and critical damping would follow a different equation solution.

Now let’s consider critical damping, when the roots are equal. This will follow a different general solution.

The discriminant is 0, so it would look like this -

Below are the initial conditions for displacement and velocity, initial time will be set to 0.

The equation above is the equation for when the discriminant is 0 and critical damping occurs.

Now we have to consider for when the discriminant is below 0, this is when you get complex conjugates are roots and the damping becomes under damping.

Below are the initial conditions for displacement and velocity, initial time will be set to 0.

So, the overall equation would follow –

Keep in mind that for the beta part, it would be the complex part of the complex number, not the complex number itself. So, the equation above doesn’t exactly work, but we could get the complex number first and then decompose it to real part and then complex part and use the equation above that to get the mathematically correct expression and result.

So here are the three equations -

Under damping –

Critical damping –

Heavy damping -